



# Balling's attenuation theory and beer composition calculator

Vilém Nohel

PMS Přerov a.s. – breweries Zubr, Litovel, Holba,  
Komenského 35, 750 02 Přerov, Czech Republic

\*corresponding author: [nohel@pivovary.cz](mailto:nohel@pivovary.cz)

## Abstract

This article deals with ways in which calculations and checks of basic parameters of beer can be performed based on the knowledge of two parameters. The importance of this method is growing with the increase in craft beer production in our country, i.e. mini breweries, home-brewing, etc. The calculations are based on Balling's equations, alcohol factors and attenuation quotients which have been published unchanged at least since 1876. In Balling's time, they were used to determine an extract with a specific gravity of 17.5 °C/17.5 °C. In current brewing, specific gravities of 20 °C/20 °C are used. For this reason, new alcohol factors of apparent attenuation were derived and based on linear regression. Regression coefficients of the dependence of the alcohol factor of apparent attenuation on the extract in original wort were calculated. Some variants of calculations based on two input parameters were incorporated into the beer composition calculator using formulas that replace extract and alcohol tables. The accuracy of the results from the calculator was checked with analyses from the Anton Paar automatic analyser and with analyses obtained by the distillation method. The calculator is available at [www.balling.cz](http://www.balling.cz).

**Keywords:** Balling's equations, alcohol factors, attenuation quotient, apparent attenuation, real attenuation, specific gravity, alcohol content, real extract, Tabarie's formula

## 1 Introduction

The basic parameters of beer include, but are not limited to: extract in original wort, apparent extract, real extract, real degree of fermentation, apparent degree of fermentation, alcohol content, specific gravities of beer, alcohol and real extract. In the case of industrial breweries, these values are usually obtained from an automatic analyser. From the perspective of craft beer production (mini-breweries, home-brewing), the purchase of an expensive device for determining the basic parameters of beer has no economic justification, and it is generally necessary to make do with a saccharometer for measurements with technology.

Professor Carl Napoleon Balling dealt with the fermentation processes of beer. Balling mathematically expressed the mass balance of wort during fermentation and experimentally verified it. This work brought him great recognition, and according to the relationships and formulas he

derived, the original concentration of wort in the brewing industry is still calculated from the determined values of alcohol and the real extract of beer (Basařová et al., 2010).

While the derivation of large Balling's formula for the calculation of the extract in original wort on the basis of mass balance is mentioned in brewing literature, Balling's alcohol factors and formulas resulting from the theory of attenuation are virtually non-existent in brewing literature today. An article by Budweiser Budvar employees in the journal *Kvasný průmysl* (Šavel et al., 2015), and an article in the *Journal of the Institute of Brewing* (Cutaia et al., 2009), are recent exceptions.

### List of symbols

$p$	extract in original wort OE (% Plato, % m/m)
$A$	alcohol A%mas (% m/m = g/100 g)
$A$	alcohol A%vol (% v/v = ml/100 ml)

$m$	apparent extract AE (% m/m)
$n$	real extract RE (% m/m)
$SG_{BEER}$	specific gravity of beer
$SG_A$	specific gravity of distillate
$SG_E$	specific gravity of residue
$ADF$	apparent degree of fermentation (%)
$RDF$	real degree of fermentation (%)
$RDF_{ASBC}$	real degree of fermentation by ASBC (%)
(ASBC	American Society of Brewing Chemists)
<i>energy</i>	energy value of beer (kcal/100 ml, kJ/100 ml)
$\rho$	density of beer (g/cm <sup>3</sup> )
$a, b, c$	alcohol factors
$q$	attenuation quotient

*Note:* Sentences and phrases in the following text written in italics in quotations marks indicate literal translation original text from historical literature.

## 2 Balling's attenuation theory

### 2.1 Balling's formulas

Balling considered the following basic quantities:

$p$  – extract in original wort

$A$  – alcohol content

$n$  – real extract

$m$  – apparent extract

The difference  $(p - m)$  expresses the apparent attenuation, which is directly proportional to the amount of fermented extract, and thus the alcohol content. Assuming that 1% m/m of the apparent attenuation is  $a$  % of alcohol, the following ratio applies:

$$\frac{a}{1} = \frac{A}{(p - m)}$$

from which  $a = \frac{A}{(p - m)}$  (1a)

Value  $a$  is called the alcohol factor of apparent attenuation. Its variable value can therefore be calculated from the known values of  $A$ ,  $p$  and  $m$ . If, on the other hand, the value of factor  $a$  is known, we can calculate the alcoholic content  $A$  at each stage of fermentation for the known  $p$  and  $m$ :

$$A = (p - m)a \quad (2a)$$

From the ratio of apparent attenuation to the original extract, the apparent degree of fermentation  $ADF$  is calculated from the following ratio:

$$\frac{p}{(p - m)} = \frac{100}{ADF}$$

$$\text{so} \quad ADF = 100 \frac{(p - m)}{p} \quad (3a)$$

We can also derive the alcohol content from the real attenuation  $(p - n)$ :

$$A = (p - n)b \quad (2b)$$

and from it the alcohol factor of real attenuation

$$b = \frac{A}{(p - n)} \quad (1b)$$

and the real degree of fermentation

$$RDF = 100 \frac{(p - n)}{p} \quad (3b)$$

The apparent attenuation is always numerically greater than the real attenuation. By subtracting the real attenuation from the apparent attenuation, the attenuation difference is obtained:

$$(p - m) - (p - n) = n - m$$

If the  $c$  amount of alcohol corresponding to 1% m/m attenuation difference, and  $A$  alcohol pertaining to the whole attenuation difference with respect to 100 g fermented wort, then

$$\frac{c}{1} = \frac{A}{(n - m)}$$

and from it  $c = \frac{A}{(n - m)}$  (1c)

$$A = (n - m)c \quad (2c)$$

Factor  $c$  is the alcohol factor of the attenuation difference.

Another significant value is the attenuation quotient  $q$ , defined as the ratio of apparent and real attenuation:

$$q = \frac{(p - m)}{(p - n)} \quad (4)$$

By dividing equations 2a and 2b, we get the following relation:

$$\frac{A}{A} = \frac{(p - m)a}{(p - n)b}$$

$$\frac{b}{a} = \frac{(p - m)}{(p - n)} = q$$

$$a = \frac{b}{q} \quad (5)$$

The alcohol factor of the attenuation difference  $c$  can also be derived from equations 2b and 2c:

$$\begin{aligned}
 A &= A \\
 c(n - m) &= b(p - n) \\
 c &= \frac{b(p - n)}{(n - m)} \\
 c &= \frac{b(p - n)}{(n - m + p - p)} \\
 c &= \frac{b(p - n)}{\frac{(p - m) - (p - n)}{(p - n)}} \\
 c &= \frac{b}{(q - 1)} \tag{6}
 \end{aligned}$$

From equation 4 we can derive a relation for calculating  $p$ :

$$\begin{aligned}
 \frac{(p - m)}{(p - n)} &= q \\
 p - m &= (p - n)q \\
 p - m &= pq - nq \tag{7a}
 \end{aligned}$$

$$\begin{aligned}
 p - pq &= m - nq \\
 p(1 - q) &= m - nq \tag{7b}
 \end{aligned}$$

$$p = \frac{(m - nq)}{(1 - q)}$$

$$p = \frac{(nq - m)}{(q - 1)} \tag{7c}$$

or equation 7b can be modified to:

$$\begin{aligned}
 p(1 - q) + nq &= m \\
 p(1 - q) + nq - n &= m - n \\
 -p(q - 1) + n(q - 1) &= m - n \\
 p(q - 1) &= n(q - 1) - m + n
 \end{aligned}$$

$$p = n + \frac{(n - m)}{(q - 1)} \tag{7d}$$

By combining equations 7d (Balling’s small formula) and 6, we receive a relation for calculating  $p$  in the following form:

$$\begin{aligned}
 p &= \frac{(n - m)}{(q - 1)} + n \quad \text{and} \quad c = \frac{b}{(q - 1)} \\
 p &= \frac{1}{(q - 1)}(n - m) + n \quad \text{and} \quad q - 1 = \frac{b}{c} \\
 p &= \frac{c}{b}(n - m) + n \tag{8}
 \end{aligned}$$

By combining equations 7a and 5, a relation can be derived for calculating the real extract  $n$ :

$$\begin{aligned}
 nq &= pq - p + m = pq - (p - m) \\
 n &= p - \frac{(p - m)}{q} \tag{9a}
 \end{aligned}$$

$$n = p - (p - m) \frac{a}{b} \tag{9b}$$

“After many, many tests and calculations, Balling calculated the alcohol factors for each wort density from 5 to 30 degrees of saccharometer” (Suk, 1880).

Figure 1 shows a table of alcohol factors and attenuation quotients for the fermentation of worts with 6–16% (Schmelzer a Skvrna, 1877), and Figure 2 shows Balling’s table for alcohol factors and attenuation quotients for fermentation of beer worts containing 6–30% of extract (Šula, 1898).

**1 I. Tabulka faktorů a attenuačních kvocientů pro kvašení mladinek 6—16 procentových.**

2 Původní síla mladinky udaná v stupních saccharometru	3 Faktory líhové pro			4 Attenuační kvocient
	* zdánlivou	** skutečnou	**** attenuační rozdíl	
	*** attenuace			
= p	= a	= b	= c	= q
6	0·4073	0·4993	2·2096	1·226
7	4091	5020	2116	227
8	4110	5047	2137	228
9	4129	5074	2160	229
10	0·4148	0·5102	2·2184	1·230
11	4167	5130	2209	231
12	4187	5158	2234	232
13	4206	5187	2262	233
14	4226	5215	2290	234
15	4246	5245	2319	235
16	4267	5274	2350	236

12

Figure 1 Alcohol factors and attenuation quotients of 6–16 % original wort from the book *Stručné pivovarnictví* (Schmelzer a Skvrna, 1877)

- 1 Table of alcohol factors and attenuation quotients for the fermentation of worts with 6–16 percent
- 2 The original wort extract given in degrees of saccharometer
- 3 Alcohol factors for \*apparent and \*\*real \*\*\*attenuation, \*\*\*\*attenuation difference
- 4 Attenuation quotient

Numerical values of alcohol factors are identical and appear subsequently in professional brewing literature up to the present, e.g. (Suk, 1880; Thausing, 1882; Lhotský, 1954; Bednář et al., 1967; Albl et al., 1990; Hardwick, 1995). In the publication *STRUČNÉ PIVOVARNICTVÍ* (Brewery industry briefly) from 1877, Schmelzer and Skvrna end their preface IN PRAGUE on the day of St. Lucie 1876, meaning that values of factors do not change at least from this year on.

**1 Tabulka Ballingova pro alkoholové faktory a attenuační quotienty pro kvašení pивních mladin 6–30% extraktu obsahujících.**

2 Procenta extraktu v původní mladině	3 Alkoholové faktory pro			4 Quotient attenuační $\frac{p-m}{p-n}$
	* attenuaci		**** rozdíl attenuační $n-m$	
	** zdánlivou $p-m$	*** skutečnou $p-n$		
= p	= a	= b	= c	= q
1	0,3983	0,4864	2,2010	1,221
2	0,4001	89	24	2
3	18	0,4915	41	3
4	36	41	58	4
5	54	67	76	5
6	73	93	96	6
7	91	0,5020	2,2116	7
8	0,4110	47	37	8
9	29	74	60	9
10	48	0,5102	84	1,230
11	67	30	2,2209	1
12	87	58	34	2
13	0,4206	87	62	3
14	26	0,5215	90	4
15	46	45	2,2319	5
16	67	74	50	6
17	88	0,5304	81	7
18	0,4309	34	2,2414	8
19	30	65	48	9
20	51	96	83	1,240
21	73	0,5427	2,2519	1
22	95	58	57	2
23	0,4417	90	95	3
24	39	0,5523	2,2636	4
25	62	55	77	5
26	85	89	2,2719	6
27	0,4508	0,5622	63	7
28	32	56	2,2808	8
29	56	90	54	9
30	80	0,5725	2,2902	1,250

**Figure 2** Alcohol factors and attenuation quotients from the book *Návod ku zkoušení surovin, produktů a odpadků pivovarnických, 1. díl Rozborů chemické* (Šula, 1898)

- 1 Balling's table for alcohol factors and attenuation quotients of fermented worts containing 6–30% of extract
- 2 Percentage of extract in original wort
- 3 Alcohol factors for \*\*apparent and \*\*\*real \*attenuation, \*\*\*\*attenuation difference
4. Attenuation quotient

## 2.2 Balling's saccharometric test

Balling introduced a saccharometric test of beer in brewing based on the attenuation difference. "After removing carbonic acid from the beer by shaking in the bottle, the apparent extract  $m$  was determined with a saccharometer or pycnometer. Then, the beer is gradually boiled down to remove all the alcohol, which makes the beer lighter; after cooling, distilled water is added to get the beer to its original weight, and the real extract  $n$  is determined with a saccharometer or pycnometer" (Schmelzer and Skvrna, 1877). From the tabulated values of alcohol factors and attenuation quotients, and using equations 7d and 2c, "the extract in original wort and the amount of alcohol are determined".

With a pycnometer, the measured values "of relative weight" were found in Balling's extract table and "saccharometric data in percentages were subtracted" for  $m$  and  $n$  (see part of Balling's extract table in Figure 3).

"To make it easier to understand and calculate the extract in original wort and to determine the amount of alcohol", Balling created tables and guidelines for their use. Figure 4 shows the table "Determining the extract in

original wort", where in the first column there is an attenuation difference ranging from 1.000 to 2.700, and in the following six columns there are values  $(n-m)/(q-1)$  for real attenuation ( $p-n$ ), "to which these differences in worts of 9–14% of the mean extract content belong". When the extract in original wort is determined according to this table, it is possible to determine the amount of alcohol according to the table in Figure 5. In the first column there are attenuation differences, and the following six columns show "the amount of alcohol" corresponding to these attenuation differences at a "wort extract ranging from 8.51 to 14.50%".

If the alcohol was determined directly by distillation, the extract in original wort was calculated according to Balling's large formula:

$$p = 100 \frac{(2.0665A + n)}{(1.0665A + 100)} \quad (10)$$

Balling's formula for calculating  $p$  was gradually revised by Holzener, Schönfeld, Doemens and Trol (Lhotský, 1954). Recently, Scandinavian authors have commented on the validity of Balling's formula (Nielsen and Kristiansen, 2007, cited Šavel et al., 2015), and details of attempts to improve the model of Balling's original equation have been discussed (Cutaia et al., 2009). Due to its universality, the formula is a master tool for evaluating the production of alcohol under various conditions, and a revision would disrupt its universality; this will not even happen in the future due to the 150 years of practice (Šavel et al., 2015).

## 2.3 Dependence of apparent attenuation on real attenuation

It can be assumed that Balling performed most of his measurements using beers with the extract in original wort that were on the market and for which he developed "tables to determine the strength of the wort and the amount of alcohol" based on the attenuation difference (see above). "10° beer was most often brewed in breweries in Bohemia" (Šula, 1898). Based on these results, he then calculated the appropriate values for the entire range of the extract in original wort ranging from 1 to 30% m/m.

As can be seen from the calculations in Tables 1 and 2, there is a linear dependence between apparent and real attenuation. Table 1 shows the calculated values of apparent attenuation ( $p-m$ ) and real attenuation ( $p-n$ ) based on the use of Balling's alcohol factors  $a$  and  $b$  and quotients  $q$  for extracts of the extract in orig-

**Table 1** Calculation of apparent extract and slope of the linear dependence of apparent attenuation on real attenuation for a specific gravity of 17.5 °C/17.5 °C

Input date from table 4		Balling's alcohol factors and quotient q			Calculated value		
p (OE)	n (RE)	a	b	q	m (AE)	Apparent attenuation	Real attenuation
						(p - m)	(p - n)
9	3.150	0.4129	0.5075	1.229	1.810	7.190	5.850
10	3.500	0.4148	0.5102	1.230	2.004	7.996	6.500
11	3.850	0.4167	0.5130	1.231	2.197	8.803	7.150
12	4.200	0.4187	0.5159	1.232	2.390	9.610	7.800
13	4.550	0.4206	0.5187	1.233	2.579	10.421	8.450
14	4.900	0.4226	0.5216	1.234	2.768	11.232	9.100
r = correlation coefficient slope intercept α = angle of the line with the positive direction of the x-axis					r = 0.9999991		
					slope = 1.2436		
					intercept = -0.0873		
					α = 51°11'48"		

inal wort ranging from 9 to 14% m/m (real extracts  $n$  at the real degree of fermentation  $RDF = 65\%$ ). Table 2 shows calculated values of apparent and real attenuation for the whole range of the extract in original wort ranging from 1 to 30% m/m.

The apparent extracts  $m$  were calculated from Balling's equation 4:

$$q = \frac{(p - m)}{(p - n)}$$

$$m = p - q(p - n) \quad (4a)$$

For the assumed measurement range for beers from 9 to 14% m/m, the slope is 1.2436 and the angle  $\alpha$  (which forms a line with the positive direction of the x-axis) is 51°11'48".

For the whole range of the original wort extract from 1 to 30% m/m, the slope is 1.2511 and the angle  $\alpha$  is 51°21'56". The difference between the angles of the two lines is minimal (0°10'08"). In practice, it is a matter of finding a line that is as close as possible to the measured values. Balling apparently did the same.

## 2.4 Calculation of Balling's alcohol factors

They are currently used to determine the extract and alcohol with specific gravities of 20 °C/20 °C. However, in Balling's time they were used to determine the extract with specific gravities of 17.5 °C/17.5 °C and to determine the alcohol content with specific gravities of 15 °C/15 °C (Schmelzer and Skvrna,

**1. Ballingova tabulka ku redukci hutnoty na stupně saccharometrické.**

* Hutnota při 14° R = 17.5° C	** Proc. ex-traktu	* Hutnota při 14° R = 17.5° C	** Proc. ex-traktu	* Hutnota při 14° R = 17.5° C	** Proc. ex-traktu	* Hutnota při 14° R = 17.5° C	** Proc. ex-traktu
1,0000	0,000	1,0051	1,275	1,0102	2,550	1,0153	3,825
1	0,025	2	1,300	3	2,575	4	3,850
2	0,050	3	1,325	4	2,600	5	3,875
3	0,075	4	1,350	5	2,625	6	3,900
4	0,100	5	1,375	6	2,650	7	3,925
5	0,125	6	1,400	7	2,675	8	3,950
6	0,150	7	1,425	8	2,700	9	3,975
7	0,175	8	1,450	9	2,725	1,0160	4,000
8	0,200	9	1,475	1,0110	2,750	1	4,025
9	0,225	1,0060	1,500	1	2,775	2	4,050
1,0010	0,250	1	1,525	2	2,800	3	4,075
1	0,275	2	1,550	3	2,825	4	4,100
2	0,300	3	1,575	4	2,850	5	4,125
3	0,325	4	1,600	5	2,875	6	4,150
4	0,350	5	1,625	6	2,900	7	4,175
5	0,375	6	1,650	7	2,925	8	4,200
6	0,400	7	1,675	8	2,950	9	4,225
7	0,425	8	1,700	9	2,975	1,0170	4,250
8	0,450	9	1,725	1,0120	3,000	1	4,275
9	0,475	1,0070	1,750	1	3,025	2	4,300
1,0020	0,500	1	1,775	2	3,050	3	4,325
1	0,525	2	1,800	3	3,075	4	4,350
2	0,550	3	1,825	4	3,100	5	4,375
3	0,575	4	1,850	5	3,125	6	4,400
4	0,600	5	1,875	6	3,150	7	4,425
5	0,625	6	1,900	7	3,175	8	4,450
6	0,650	7	1,925	8	3,200	9	4,475
7	0,675	8	1,950	9	3,225	1,0180	4,500
8	0,700	9	1,975	1,0130	3,250	1	4,525
9	0,725	1,0080	2,000	1	3,275	2	4,550
1,0030	0,750	1	2,025	2	3,300	3	4,575
1	0,775	2	2,050	3	3,325	4	4,600
2	0,800	3	2,075	4	3,350	5	4,625
3	0,825	4	2,100	5	3,375	6	4,650
4	0,850	5	2,125	6	3,400	7	4,675
5	0,875	6	2,150	7	3,425	8	4,700
6	0,900	7	2,175	8	3,450	9	4,725
7	0,925	8	2,200	9	3,475	1,0190	4,750
8	0,950	9	2,225	1,0140	3,500	1	4,775
9	0,975	1,0090	2,250	1	3,525	2	4,800
1,0040	1,000	1	2,275	2	3,550	3	4,825
1	1,025	2	2,300	3	3,575	4	4,850
2	1,050	3	2,325	4	3,600	5	4,875
3	1,075	4	2,350	5	3,625	6	4,900
4	1,100	5	2,375	6	3,650	7	4,925
5	1,125	6	2,400	7	3,675	8	4,950
6	1,150	7	2,425	8	3,700	9	4,975
7	1,175	8	2,450	9	3,725	1,0200	5,000
8	1,200	9	2,475	1,0150	3,750	1	5,025
9	1,225	1,0100	2,500	1	3,775	2	5,050
1,0050	1,250	1	2,525	2	3,800	3	5,075

**Figure 3** Title page of Balling's table for converting density to % extract from the book *Návod ku zkoušení surovin, produktů a odpadků pivovarnických, 1. díl Rozborů chemické (Šula, 1898)*

1 Balling's table of relative density reduction into saccharometric degrees  
\*Relative density at, \*\*Percentage of extract

**Table 2** Calculation of apparent extract and slope of the linear dependence of apparent attenuation on real attenuation for a specific gravity of 17.5 °C/17.5 °C

Input date from table 4		Balling's alcohol factors and quotient q			Calculated value		
p (OE)	n (RE)	a	b	q	m (AE)	Apparent attenuation	Real attenuation
						(p - m)	(p - n)
1	0.350	0.3983	0.4864	1.221	0.206	0.794	0.650
2	0.700	0.4001	0.4890	1.222	0.411	1.589	1.300
3	1.050	0.4018	0.4915	1.223	0.615	2.385	1.950
4	1.400	0.4036	0.4941	1.224	0.817	3.183	2.600
5	1.750	0.4054	0.4967	1.225	1.018	3.982	3.250
6	2.100	0.4073	0.4994	1.226	1.218	4.782	3.900
7	2.450	0.4091	0.5020	1.227	1.416	5.584	4.550
8	2.800	0.4110	0.5047	1.228	1.614	6.386	5.200
9	3.150	0.4129	0.5075	1.229	1.810	7.190	5.850
10	3.500	0.4148	0.5102	1.230	2.004	7.996	6.500
11	3.850	0.4167	0.5130	1.231	2.197	8.803	7.150
12	4.200	0.4187	0.5159	1.232	2.390	9.610	7.800
13	4.550	0.4206	0.5187	1.233	2.579	10.421	8.450
14	4.900	0.4226	0.5216	1.234	2.768	11.232	9.100
15	5.250	0.4246	0.5245	1.235	2.956	12.044	9.750
16	5.600	0.4267	0.5275	1.236	3.144	12.856	10.400
17	5.950	0.4288	0.5304	1.237	3.331	13.669	11.050
18	6.300	0.4309	0.5335	1.238	3.515	14.485	11.700
19	6.650	0.4330	0.5365	1.239	3.697	15.303	12.350
20	7.000	0.4351	0.5396	1.240	3.878	16.122	13.000
21	7.350	0.4373	0.5427	1.241	4.059	16.941	13.650
22	7.700	0.4395	0.5459	1.242	4.238	17.762	14.300
23	8.050	0.4417	0.5491	1.243	4.415	18.585	14.950
24	8.400	0.4439	0.5523	1.244	4.590	19.410	15.600
25	8.750	0.4462	0.5556	1.245	4.766	20.234	16.250
26	9.100	0.4485	0.5589	1.246	4.940	21.060	16.900
27	9.450	0.4508	0.5623	1.247	5.111	21.889	17.550
28	9.800	0.4532	0.5656	1.248	5.284	22.716	18.200
29	10.150	0.4556	0.5691	1.249	5.455	23.545	18.850
30	10.500	0.4580	0.5726	1.250	5.623	24.377	19.500
r = correlation coefficient slope intercept α = angle of the line with the positive direction of the x-axis					r = 0.99998		
					slope = 1.2511		
					intercept = -0.1074		
					α = 51°21'56"		

1877; Šula, 1898). For this reason, the use of Balling's alcohol factors of apparent attenuation a in the current brewing industry would not lead to completely accurate results.

For illustration, Table 3 shows values for specific gravities of 20 °C/20 °C and for specific gravities of 17.5 °C/17.5 °C for the extract in original wort of 11%

m/m from the relevant tables and a real degree of fermentation of 65%. To calculate the specific gravity of beer 17.5 °C/17.5 °C, the formula according to Tabarie was used, which was also used by Balling at the time (Šavel et al., 2015). An improved Tabarie's formula was used to calculate the specific gravity of beer at 20 °C/20 °C (Nielsen and Aastrup, 2004).

Alcohol factors *a* and *b* were calculated according to equations 1a and 1b, and attenuation quotients *q* according to equation 4. For a specific gravity of 17.5 °C/17.5 °C, we obtain identical values of the alcohol factors *a*, *b* and the attenuation quotient *q* with the values derived by Balling.

Due to the different numerical values of the apparent extracts *m*, there will also be different numerical values of the quotients *q*. Since the alcohol factor of real attenuation *b* is constant for a given *p* (does not change with fermentation) and is equal to the product of *aq* (*a* times *q*), the numerical values of alcohol factors *a* will also vary.

To determine the alcohol factors for specific gravities of 20 °C/20 °C, calculations were performed in an EXCEL spreadsheet for each extract in original wort from 1 to 30% m/m for real degree of fermentation 65% (Table 4).

Calculation principle: formula 3b for real fermentation was used to calculate the real extracts *n*, and Balling's large formula 10 was used to calculate the alcohol contents *A%mas*.

The respective values of specific gravities were subtracted directly by interpolation from the tables used in the Anton Paar automatic analyzer. For the extract „the extract according to ASBC tables“ (ASBC, 2020); for alcohol from the annex to the guidebook Alcolyzer Plus “Density and Concentration of Water/ Ethanol Solutions” (Anton Paar, © 2007).

The specific gravities of beers were calculated according to the improved Tabarie's formula. Tabarie's formula is as follows:

$$SG_{BEER} = SG_E + SG_A - 1 \tag{11}$$

According to Scandinavian authors (Nielsen and Aastrup, 2004), the accuracy of Tabarie's formula is problematic – especially for strong beer. Based on many analyses of beer performed in Alfred Jørgensen's laboratory, an improved version of the old Tabarie's formula is proposed:

$$SG_{BEER} = SG_E + SG_A - 1 + 2.96(1 - SG_A)^2 - 381(1 - SG_A)^3 \tag{12}$$

**Table 3** Alcohol factors and quotient *q* for *p* = 11% m/m and for specific gravities of 20 °C/20 °C and 17.5 °C/17.5 °C

Specific gravity	<i>p</i> (OE)	RDF	<i>n</i> (RE)	<i>SG<sub>E</sub></i>	<i>A%mas</i>	<i>SG<sub>A</sub></i>	<i>SG<sub>BEER</sub></i>	<i>m</i> (AE)	ADF	<i>a</i>	<i>b</i>	<i>q</i>
20 °C/20 °C	11	65	3.85	1.01510	3.67	0.99337	1.00849	2.175	80.22	0.4157	0.5130	1.234
17,5 °C/17,5 °C	11	65	3.85	1.01540	3.67	0.99338	1.00878	2.196	80.04	0.4166	0.5130	1.231

178 1 Tabulka II. Určení původní síly mladinky.

2 Attenuační rozdíl = <i>n</i> - <i>m</i>	3 Tato příslušící hodnota <i>p</i> - <i>n</i> při původní síle mladinky v procentech					
	9 8:51 až 9:5	10 9:51 až 10:5	11 10:51 až 11:5	12 11:51 až 12:5	13 12:51 až 13:5	14 13:51 až 14:5
1:000	4:266	4:347	4:329	4:310	4:292	4:278
25	475	455	437	418	399	379
50	584	564	545	525	506	486
75	693	672	653	633	613	593
1:100	802	781	761	741	721	700
25	901	890	870	849	828	807
50	5:020	4:998	4:978	4:956	4:935	4:913
75	130	5:107	5:086	5:064	5:043	5:020
1:200	239	216	194	172	150	127
25	348	324	302	280	258	234
50	457	433	411	387	365	341
75	566	542	519	495	472	447
1:300	676	650	627	603	580	554
25	785	759	735	711	687	661
50	894	868	843	818	794	768
75	6:003	5:976	5:952	5:926	5:901	5:875
1:400	112	6:085	6:060	6:034	6:009	5:981
25	221	194	168	142	116	6:088
50	331	302	276	249	223	195
75	440	411	384	357	331	302
1:500	549	520	493	465	438	409
25	659	630	601	573	545	516
50	768	739	709	680	653	623
75	877	847	817	788	760	729
1:600	986	956	925	896	867	836
25	7:096	7:065	7:034	7:004	6:974	6:943
50	205	173	142	112	7:082	7:050
75	314	282	250	219	189	157
1:700	423	391	358	327	296	264
25	532	500	466	435	404	371
50	608	608	575	543	511	478
75	717	717	685	650	618	585
1:800	826	791	758	726	691	661
25	934	899	866	833	798	768
50	8:043	8:007	7:974	7:940	7:905	7:870
75	152	116	8:081	8:047	8:012	8:012
1:900	260	225	189	155	119	119
25	333	333	297	262	226	226
50	441	441	405	369	333	333
75	549	549	512	477	440	440
2:000	658	620	584	547	510	473
25	766	728	691	653	616	579
50	874	836	798	760	723	685
75	982	943	905	867	829	791
2:100	9:090	9:051	9:012	9:012	9:012	9:012
25	159	159	120	120	120	120
50	267	267	227	227	227	227
75	374	374	334	334	334	334
2:200	482	442	401	401	401	401
2:300	913	871	828	828	828	828
2:400	10:300	10:255	10:210	10:210	10:210	10:210
2:500	729	729	683	683	683	683
2:600	11:111	11:111	11:111	11:111	11:111	11:111
2:700	537	537	537	537	537	537

**Figure 4** Table of determining the extract in original wort from the book *Stručně pivovarnictví* (Schmelzer a Skvrna, 1877)

- 1 Table II. Extract determination of original wort
- 2 Attenuation difference
- 3 This corresponds to the value (*p* - *n*) of the original extract expressed as a percentage

While the old Tabarie's formula is less accurate for strong beer than for medium-strength beer, the improved version is valid for all beers up to *A%mas* = 8, unless the degree of fermentation is extreme. The real degree of fermentation should be 65 to 85%. Outside this range, accuracy has not been checked and increasing deviations can be expected (Nielsen and Aastrup, 2004).

**Table 4** Calculation of Balling's alcohol factors and attenuation quotients of 1–30% m/m original wort for a specific gravity of 20 °C/20 °C

p (OE)	RDF	n (RE)	SG <sub>E</sub>	A%-mas	SG <sub>A</sub>	SG <sub>BEER</sub>	m (AE)	ADF	a	b	c	q	
												$\frac{b}{a}$	$\frac{(p-m)}{(p-n)}$
1	65	0.350	1.00136	0.32	0.99939	1.00075	0.193	80.70	0.3918	0.4864	2.0138	1.242	1.242
2	65	0.700	1.00273	0.64	0.99879	1.00152	0.391	80.46	0.3950	0.4890	2.0558	1.238	1.238
3	65	1.050	1.00409	0.96	0.99818	1.00228	0.586	80.47	0.3970	0.4915	2.0648	1.238	1.238
4	65	1.400	1.00546	1.28	0.99758	1.00304	0.780	80.49	0.3990	0.4941	2.0734	1.238	1.238
5	65	1.750	1.00682	1.61	0.99697	1.00381	0.978	80.45	0.4013	0.4967	2.0901	1.238	1.238
6	65	2.100	1.00819	1.95	0.99637	1.00458	1.175	80.41	0.4037	0.4994	2.1059	1.237	1.237
7	65	2.450	1.00957	2.28	0.99577	1.00536	1.375	80.36	0.4061	0.5020	2.1242	1.236	1.236
8	65	2.800	1.01095	2.62	0.99517	1.00614	1.575	80.31	0.4085	0.5047	2.1433	1.236	1.236
9	65	3.150	1.01233	2.97	0.99457	1.00692	1.774	80.29	0.4109	0.5075	2.1578	1.235	1.235
10	65	3.500	1.01371	3.32	0.99397	1.00770	1.973	80.27	0.4132	0.5102	2.1720	1.235	1.235
11	65	3.850	1.01510	3.67	0.99337	1.00849	2.175	80.22	0.4157	0.5130	2.1905	1.234	1.234
12	65	4.200	1.01649	4.02	0.99277	1.00927	2.374	80.22	0.4180	0.5159	2.2033	1.234	1.234
13	65	4.550	1.01788	4.38	0.99217	1.01005	2.572	80.22	0.4203	0.5187	2.2159	1.234	1.234
14	65	4.900	1.01928	4.75	0.99157	1.01083	2.771	80.21	0.4227	0.5216	2.2293	1.234	1.234
15	65	5.250	1.02068	5.11	0.99097	1.01161	2.969	80.21	0.4251	0.5245	2.2416	1.234	1.234
16	65	5.600	1.02208	5.49	0.99038	1.01239	3.166	80.21	0.4274	0.5275	2.2541	1.234	1.234
17	65	5.950	1.02349	5.86	0.98978	1.01317	3.363	80.22	0.4298	0.5304	2.2659	1.234	1.234
18	65	6.300	1.02490	6.24	0.98918	1.01394	3.558	80.23	0.4322	0.5335	2.2766	1.234	1.234
19	65	6.650	1.02632	6.63	0.98858	1.01472	3.755	80.24	0.4346	0.5365	2.2886	1.234	1.234
20	65	7.000	1.02773	7.01	0.98799	1.01549	3.949	80.26	0.4370	0.5396	2.2989	1.235	1.235
21	65	7.350	1.02915	7.41	0.98739	1.01625	4.140	80.29	0.4394	0.5427	2.3079	1.235	1.235
22	65	7.700	1.03058	7.81	0.98679	1.01701	4.331	80.31	0.4418	0.5459	2.3174	1.236	1.236
23	65	8.050	1.03201	8.21	0.98619	1.01776	4.519	80.35	0.4442	0.5491	2.3251	1.236	1.236
24	65	8.400	1.03344	8.62	0.98560	1.01851	4.708	80.39	0.4466	0.5523	2.3335	1.237	1.237
25	65	8.750	1.03488	9.03	0.98500	1.01926	4.895	80.42	0.4491	0.5556	2.3422	1.237	1.237
26	65	9.100	1.03631	9.45	0.98440	1.01999	5.078	80.47	0.4515	0.5589	2.3482	1.238	1.238
27	65	9.450	1.03775	9.87	0.98378	1.02069	5.253	80.55	0.4537	0.5623	2.3509	1.239	1.239
28	65	9.800	1.03920	10.29	0.98318	1.02141	5.433	80.60	0.4562	0.5656	2.3572	1.240	1.240
29	65	10.150	1.04065	10.73	0.98258	1.02211	5.607	80.66	0.4586	0.5691	2.3615	1.241	1.241
30	65	10.500	1.04210	11.16	0.98197	1.02281	5.781	80.73	0.4610	0.5726	2.3661	1.242	1.242

According to our findings, the formula provides acceptable values of relative densities even for Czech beers (extract in original wort 10 to 12% m/m), which have a real degree of fermentation lower than 65%.

Apparent degrees of fermentation were calculated according to formula 3a.

Alcohol factors  $a$ ,  $b$ ,  $c$  were calculated according to equations 1a, 1b, 1c, and the attenuation quotient  $q$  was calculated according to equations 4 and 5.

The dependence of alcohol factors  $b$  on extracts in original wort  $p$  was derived by combining equations 1b and 10:

$$b = \frac{A}{(p-n)}$$

$A$  is determined from Balling's large formula:

$$A = 100 \frac{(p-n)}{(206.65 - 1.0665p)} \quad (10a)$$

and introduced into the first equation:

$$b = 100 \frac{(p-n)}{(206.65 - 1.0665p)(p-n)}$$

$$b = \frac{1}{(2.0665 - 0.010665p)} \quad (13)$$

The dependence of alcohol factors  $a$  on the extracts in original wort  $p$  was derived on the basis of linear regression, because the correlation coefficient of the depend-



**Table 5** Regression equation of the dependence of the alcohol factor "a" on the original extract (1–30 % m/m) for a specific gravity of 20 °C/20 °C:  $a = B + M \cdot p$ 

Alcohol factor	Alcohol factor "a" from table 4		
	Correlation coefficient	Regression coefficients	
<i>a</i>	<i>r</i>	<i>B</i>	<i>M</i>
(-)	0.99997	0.3895408891	0.002376648
$a = 0.389541 + 0.002377 \cdot p$			

ence of *a* on *p* is significant ( $r = 0.99997$ ). Regression coefficients of the dependence of the alcohol factor *a* on the extract in original wort *p* were calculated (Table 5). The equation has the following form:

$$a = 0.389541 + 0.002377p \quad (14)$$

Table 6 shows calculations of the values of alcohol factors *a* for  $p = 1$  to 30% m/m according to regression equation 14 and comparisons with the values of alcohol factors *a* calculated in Table 4 according to equation 1a. The differences are minimal, i.e. 0.0001 for the extract in original wort of Czech beers. Table 6 also shows values of apparent and real attenuation.

For the whole range of the extract in original wort ranging from 1 to 30% m/m, the slope of the dependence of apparent attenuation on real attenuation for specific gravities of 20 °C/20 °C is 1.2393, and the angle  $\alpha$  is 51°05'56".

The difference between the angle for 20 °C/20 °C specific gravities and the angle for 17.5 °C/17.5 °C specific gravities (Table 2) is 00°16'00".

A graphic illustration is shown in Figure 6.

### 3 Calculation of beer composition based on two input values

Balling's attenuation theory shows the relationship between the basic parameters of beer cited in the introduction. If two parameters are known, the rest can be calculated.

For some calculation alternatives, a procedure was compiled where the individual variants of the input parameter pairs are in rows from A to M, and the individual calculated beer parameters are in columns from 1 to 15. The listed calculations are incorporated into a spreadsheet, which we called the BEER COMPOSITION CALCULATOR (BCC). The calculator is available for practical use at [www.balling.cz](http://www.balling.cz). In addition to the basic cited parameters, the calculation of alcohol factors *a* and *b*, the attenuation quotient *q*, the real degree of fermentation  $RDF_{ASBC}$ , energy values (kcal/100 ml and kJ/100 ml) and beer sample density ( $g/cm^3$ ) are added.

Extract and alcohol tables are replaced in the calculator by polynomial formulas. The formulas were taken from domestic (Šavel, 1989) and foreign literature (Cutaia et al., 2009).

Formulas for calculating alcohol *A%vol*, the real degree of fermentation according to ASBC, the energy

180 1 Tabulka III. Určení množství lihu.							
2 Attenuační rozdíl = n-m	3 Temné přínáležití množství lihu při původní síle mladiny v procentech						
	9 8:51 až 9:5	10 9:51 až 10:5	11 10:51 až 11:5	12 11:51 až 12:5	13 12:51 až 13:5	14 13:51 až 14:5	
1 000	2:216	2:218	2:220	2:223	2:226	2:229	
25	271	273	276	278	282	285	
50	326	329	331	334	336	341	
75	382	384	386	390	392	395	
1:100	437	440	442	445	448	451	
25	493	495	498	501	504	507	
50	548	551	554	557	560	563	
75	603	606	609	612	615	618	
1:200	659	662	664	668	671	671	
25	714	717	720	724	727	730	
50	770	773	776	780	783	786	
75	825	828	831	834	835	841	
1:300	880	883	887	890	894	897	
25	936	939	942	946	950	953	
50	991	995	997	3:001	3:006	3:009	
75	3:047	3:051	3:053	0:57	0:61	0:65	
1:400	102	106	109	112	116	120	
25	157	162	164	167	172	176	
50	213	217	220	223	228	232	
75	269	273	276	279	284	288	
1:500	325	328	331	335	339	343	
25	380	384	385	390	395	398	
50	435	439	442	446	451	454	
75	490	495	498	502	507	510	
1:600	546	551	553	557	561	566	
25	601	606	608	613	617	621	
50	656	662	664	669	673	677	
75	711	717	719	724	729	733	
1:700	767	771	775	779	784	789	
25	842	826	831	835	840	844	
50		881	886	891	896	900	
75		937	942	946	952	956	
1:800		993	997	4:002	4:007	4:012	
25		4:048	4:053	0:58	0:63	0:67	
50		104	109	113	119	123	
75		159	164	169	174	179	
1:900		214	219	224	229	235	
25			275	279	285	289	
50			330	335	341	346	
75			386	396	397	402	
2:000			441	446	452	458	
25			497	501	507	513	
50			552	557	563	569	
75			608	612	619	625	
2:100			663	668	675	680	
25				724	731	736	
50				780	786	791	
75				835	841	847	
2:200				891	897	903	
2:300				5:113	5:119	5:126	
2:400					342	349	
2:500					605	572	
2:600						795	
2:700						6:018	

**Figure 5** Table of determining the amount of alcohol from the book *Stručné pivovarnictví (Schmelzer a Skvrna, 1877)*

- 1 Table III. Determining of the ethanol content
- 2 Attenuation difference
- 3 Corresponding alcohol content at following values of the original wort extract

value and the density of the beer sample were taken from the reference manual of Anton Paar (© 2007). Other necessary calculation formulas are the subject of this article.

Calculated alcohol content in variant A (input  $p$  and  $m$  in craft beer production) based on equation 2a using the newly calculated alcohol factors  $a$  for apparent attenuation at specific gravities of 20 °C/20 °C (Tables 4, 5 and 6) can be compared with the calculated alcohol content according to formulas published in foreign literature.

Column 4a shows the calculation of alcohol content according to the following formula:

$$A\%mas = 0.38726(OE - AE) + 0.00307(OE - AE)^2 \quad (15) \text{ (Cutaia et al., 2009)}$$

This equation was derived on the basis of a stepwise regression technique in order to find a more modern and accurate procedure for determining the prediction assumption of  $A\%mas$  compared to Balling's equation  $A = a(p - m)$ . However, by comparing the results of the calculated alcohol contents from both formulas, i.e. from the BEER COMPOSITION CALCULATOR (variant A) and according to formula 15, we find that we obtain virtually identical values (the difference for Czech beers depending on fermentation is + 0.00 to 0.03% m/m).

Similarly, in column 5 the calculation of the alcohol content  $A\%vol$  is according to "The Craft Formula":

$$A\%vol = \frac{(OE - AE)}{(2.048 - 0.0126OE)} \quad (16) \text{ (Nielsen, 2004)}$$

By comparing the results of the calculated alcohol contents from both formulas, i.e. from the BEER COMPOSITION CALCULATOR (variant A) and according to formula 16, we find a difference for Czech beers ranging from +0.04 to 0.07% v/v. In general, formulas containing  $A\%vol$  are less accurate than formulas containing  $A\%mas$ .

Column 4b shows the control calculation of the alcohol content according to formula 2b (with the alcohol factor of the real attenuation  $b$ ), the result must be identical to the result in column 4a.

#### 4 Practical verification of beer calculator accuracy

The accuracy of calculations according to the calculator was confronted with the results of analyses from Anton Paar analyzers from the database of breweries Zubr, Litovel and Holba according to variant L (input values  $A\%mas$  and

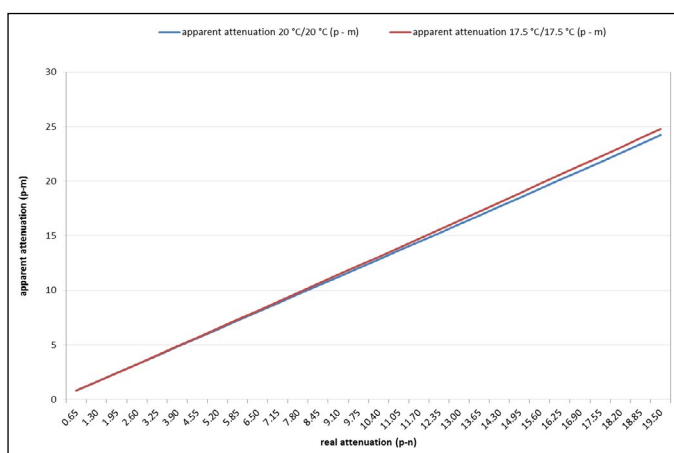


Figure 6 Dependence of apparent attenuation on real attenuation

$SG_{BEER}$ ) for Czech beers with an extract in original wort of 10 to 12% m/m. The calculated values of  $n$  and  $p$  differed from the values found by the analyzers by 0.02 to 0.04% m/m.

From the results of analyses using the Anton Paar analyzer, we can calculate attenuation quotients  $q$  as a share of apparent attenuation ( $p - m$ ) and real attenuation ( $p - n$ ). For Czech beers we get approximate values of 1.239. This corresponds to the calculated value of the slope of the apparent attenuation versus real attenuation for specific gravities of 20 °C/20 °C and for a range of  $p$  from 1 to 30% m/m (Table 6).

Table 7 illustrates the results of analyses provided by RIBM (Research Institute of Brewing and Malting in Prague) from both the Anton Paar analyzer and the distillation method. Calculations are performed for both variants using the BEER COMPOSITION CALCULATOR. For comparison with the results from the Anton Paar analyzer, the input values are  $A\%mas$  and  $SG_{BEER}$  (variant L), and for comparison with the results of the distillation method, the input values are  $SG_E$  and  $SG_A$  (variant I). The difference found for the parameters  $n$  and  $p$  when the results from the beer composition calculator and the Anton Paar analyzer were compared (0.01 to 0.04% m/m) is given by different attenuation quotients  $q$ . Compared to the value  $q = 1.239$  from the Anton Paar analyzer, for each  $p$  the calculator calculates the corresponding value of  $q$ ; for Czech beers (extract in original wort 10 to 12% m/m) these values are 1.234 to 1.235 (Table 6).

#### 5 Practical use

In recent years, several dozen new microbreweries have been put into operation in the Czech Republic every year, and there is also a growing unprecedented interest in homebrewing. In general, only a saccharometer

**Table 6** Calculation of the slope of the dependence of apparent attenuation on real attenuation for a specific gravity of 20 °C/20 °C by a linear regression

p (OE)	Calculated values from table 4				Linear regression "a"			Calculating "b"			Apparent attenuation	Real attenuation	Attenuation quotient	
	A% mas	n (RE)	m (AE)	a	b	a	$\Delta a$	A% mas	b	$\Delta b$				A% mas
						$a = 0,389541 + 0,002377 \cdot p$				$b = 1/(2,0665 - 0,010665 \cdot p)$				
1	0.32	0.350	0.193	0.3918	0.4864	0.3919	0.0001	0.32	0.4864	0.0000	0.32	0.81	0.65	1.242
2	0.64	0.700	0.391	0.3950	0.4890	0.3943	-0.0007	0.63	0.4890	0.0000	0.64	1.61	1.30	1.238
3	0.96	1.050	0.586	0.3970	0.4915	0.3967	-0.0003	0.96	0.4915	0.0000	0.96	2.41	1.95	1.238
4	1.28	1.400	0.780	0.3990	0.4941	0.3990	0.0000	1.28	0.4941	0.0000	1.28	3.22	2.60	1.238
5	1.61	1.750	0.978	0.4013	0.4967	0.4014	0.0001	1.61	0.4967	0.0000	1.61	4.02	3.25	1.238
6	1.95	2.100	1.175	0.4037	0.4994	0.4038	0.0001	1.95	0.4994	0.0000	1.95	4.82	3.90	1.237
7	2.28	2.450	1.375	0.4061	0.5020	0.4062	0.0001	2.28	0.5020	0.0000	2.28	5.63	4.55	1.236
8	2.62	2.800	1.575	0.4085	0.5047	0.4086	0.0000	2.62	0.5047	0.0000	2.62	6.42	5.20	1.236
9	2.97	3.150	1.774	0.4109	0.5075	0.4109	0.0001	2.97	0.5075	0.0000	2.97	7.23	5.85	1.235
10	3.32	3.500	1.973	0.4132	0.5102	0.4133	0.0001	3.32	0.5102	0.0000	3.32	8.03	6.50	1.235
11	3.67	3.850	2.175	0.4157	0.5130	0.4157	0.0000	3.67	0.5130	0.0000	3.67	8.82	7.15	1.234
12	4.02	4.200	2.374	0.4180	0.5159	0.4181	0.0001	4.02	0.5159	0.0000	4.02	9.63	7.80	1.234
13	4.38	4.550	2.572	0.4203	0.5187	0.4204	0.0001	4.38	0.5187	0.0000	4.38	10.43	8.45	1.234
14	4.75	4.900	2.771	0.4227	0.5216	0.4228	0.0001	4.75	0.5216	0.0000	4.75	11.23	9.10	1.234
15	5.11	5.250	2.969	0.4251	0.5245	0.4252	0.0001	5.12	0.5245	0.0000	5.11	12.03	9.75	1.234
16	5.49	5.600	3.166	0.4274	0.5275	0.4276	0.0001	5.49	0.5275	0.0000	5.49	12.83	10.40	1.234
17	5.86	5.950	3.363	0.4298	0.5304	0.4299	0.0001	5.86	0.5304	0.0000	5.86	13.64	11.05	1.234
18	6.24	6.300	3.558	0.4322	0.5335	0.4323	0.0001	6.24	0.5335	0.0000	6.24	14.44	11.70	1.234
19	6.63	6.650	3.755	0.4346	0.5365	0.4347	0.0001	6.63	0.5365	0.0000	6.63	15.25	12.35	1.234
20	7.01	7.000	3.949	0.4370	0.5396	0.4371	0.0000	7.02	0.5396	0.0000	7.01	16.05	13.00	1.235
21	7.41	7.350	4.140	0.4394	0.5427	0.4395	0.0001	7.41	0.5427	0.0000	7.41	16.86	13.65	1.235
22	7.81	7.700	4.331	0.4418	0.5459	0.4418	0.0000	7.81	0.5459	0.0000	7.81	17.67	14.30	1.236
23	8.21	8.050	4.519	0.4442	0.5491	0.4442	0.0000	8.21	0.5491	0.0000	8.21	18.48	14.95	1.236
24	8.62	8.400	4.708	0.4466	0.5523	0.4466	0.0000	8.62	0.5523	0.0000	8.62	19.29	15.60	1.237
25	9.03	8.750	4.895	0.4491	0.5556	0.4490	-0.0001	9.03	0.5556	0.0000	9.03	20.10	16.25	1.237
26	9.45	9.100	5.078	0.4515	0.5589	0.4513	-0.0001	9.44	0.5589	0.0000	9.45	20.92	16.90	1.238
27	9.87	9.450	5.253	0.4537	0.5623	0.4537	0.0000	9.87	0.5623	0.0000	9.87	21.75	17.55	1.239
28	10.29	9.800	5.433	0.4562	0.5656	0.4561	-0.0001	10.29	0.5656	0.0000	10.29	22.57	18.20	1.240
29	10.73	10.150	5.607	0.4586	0.5691	0.4585	-0.0001	10.72	0.5691	0.0000	10.73	23.39	18.85	1.241
30	11.16	10.500	5.781	0.4610	0.5726	0.4608	-0.0002	11.16	0.5726	0.0000	11.16	24.22	19.50	1.242
r = correlation coefficient													r = 0.999992	
slope													slope = 1.2393	
intercept													intercept = -0.0256	
$\alpha$ = angle of the line with the positive direction of the x-axis													$\alpha = 51^{\circ}05'56''$	

is used to check the technology. In cases where the final beer was prepared from one batch of wort, other beer parameters can be calculated based on the measurements of  $p$  and  $m$  (variant A).

Calculations using the beer composition calculator are very accurate. In order for the calculated beer parameters to correspond to reality, the input of the correct  $p$  and  $m$  values must be ensured. Based on experience from operational measurements in our research mini-brewery in the Litovel brewery, it is recommended to take into account the water with brewer's yeast for the parameter  $p$ , which is why the value found in the fermentation cellar is introduced immediately after pitching (not in the brewhouse). When measuring the apparent extract  $m$  with a saccharometer, if the beer is not free of  $\text{CO}_2$  (this can also be ensured by manual shaking (Basařová, 1993)), the  $\text{CO}_2$  bubbles lighten the saccharometer, and this creates an error in comparison with the measurement in beer freed of  $\text{CO}_2$  by shaking after previous heating to a temperature close to 20 °C (Hlaváček and Lhotský, 1972). Table 8 shows an example of deriving an acceptable correction for  $\text{CO}_2$  for the apparent extract when measured with a saccharometer (0.2% m/m).

Variant C or D can be used to calculate the minimum and maximum values of beer parameters for individual groups (draught, lager) or tax zones (e.g. for  $p = 11.00$  to 11.99% m/m), which are required for audits of certified quality management systems. Brewing parameters, including fermentation, generally follow a normal (Gaussian) distribution. Using the mean ( $\mu$ ) and the standard deviation ( $\sigma$ ) in the interval  $\mu \pm 3\sigma$ , we calculate the required limit values.

Variants G, H and I can be used in laboratories to check the accuracy of measuring three specific gravities –  $SG_p$ ,  $SG_A$  and  $SG_{BEER}$  – with a pycnometric distillation method.

## 6 Conclusion

Professor Balling excellently derived the theory of attenuation, which is of immense importance to brewing. His formula for calculating the extract in original wort from specified values of the real extract and alcohol in finished beer is used worldwide.

He introduced important quantities for the fermentation process such as apparent and real attenuation, apparent and real degree of fermentation, alcohol factors and the attenuation quotient.

Alcohol factors and attenuation quotients derived by Balling in the mid-19th century have been published unchanged since at least 1876. Specific gravities of 17.5 °C/17.5 °C were used to determine the extract, and specific gravities of 15 °C/15 °C were used to determine

the alcohol content. Balling's small formula can be used to calculate the values of apparent extracts  $m$  from the tabulated values of attenuation quotients  $q$  for known values of the extract in original wort  $p$  and the real extract  $n$ . For a range of  $p$  from 1 to 30% m/m and a specific gravity of 17.5 °C/17.5 °C, a linear dependence of the apparent attenuation on real attenuation ( $r = 0.999998$ ), which is characterized by a slope of 1.2511 and an angle of  $\alpha = 51^\circ 21' 56''$ , can be deduced.

In today's brewing, the values of extract and alcohol are determined from specific gravities of 20 °C/20 °C. An improved Tabarie's formula was used to calculate the specific gravities of beer to determine the dependence of apparent attenuation on real attenuation. For a range of  $p$  from 1 to 30% m/m, a linear dependence of the apparent attenuation on the real attenuation ( $r = 0.999992$ ), which is characterized by a slope of 1.2393 and an angle of  $\alpha = 51^\circ 05' 56''$ , was deduced.

The numerical difference of values of the slopes of both lines, or their angles ( $0^\circ 16' 0''$ ), is due to different values of attenuation quotients  $q$  (for different relative densities of 17.5 °C/17.5 °C and 20 °C/20 °C) and alcohol factors of apparent attenuation  $a$  for the given  $p$ , as the alcohol factor of real attenuation  $b$  for the given  $p$  is constant and equal to the product of  $aq$  ( $a$  times  $q$ ) (the  $q$  changed, so the  $a$  must also change so that their product remains the same).

To use Balling's alcohol factors in current brewing, new alcohol factors of apparent attenuation  $a$  were calculated for a range of  $p$  from 1 to 30% m/m. A formula was derived based on linear regression to calculate the alcohol factors  $a$ :

$$a = 0.389541 + 0.002377p$$

Alcohol factors of real attenuation  $b$  are derived from the equation  $b = A/(p - n)$ , where a transformed Balling's large formula is introduced for  $A [A = 100(p - n)/(206.65 - 1.0665p)]$ ; after modification, the equation has the following form:  $b = 1/(2.0665 - 0.010665p)$

The calculated values of the alcohol factors of real attenuation  $b$  are identical to the alcohol factors derived by Balling.

The spreadsheet contains some variants of beer composition calculations based on the inputs of two parameters (BEER COMPOSITION CALCULATOR). The calculator results can be exported in PDF format. The calculator can be used by practice of mini-breweries or home brewing to determine the composition of beer, in laboratories to check the accuracy of beer analysis by the reference (distillation) method and also in teaching in brewing schools.

**Table 7** Comparison of beer analysis results from an Anton Paar analyzer and the distillation method with the results of calculation with the calculator (BCC)

Beer	RIBM – beer from Anton Paar analyzer BCC – variant L						RIBM – beer by distillation BCC – variant I						$\Delta p$ distillation – Anton Paar	
	Source of calculation	A% mas.	A% vol.	n (RE)	$SG_{BEER}$	p (OE)	Source of calculation	A% mas.	A% vol.	n (RE)	$SG_E$	$SG_A$		p (OE)
1	RIBM		0.23	6.31		6.67	RIBM	0.21	0.27	6.27			6.69	0.02
	BCC	0.18	0.23	6.30	1.02457	6.66	BCC	0.21	0.28	6.29	1.02486	0.99962	6.71	0.05
	$\Delta$ RIBM – BCC		0	0.01		0.01	$\Delta$ RIBM – BCC	0	-0.01	-0.02			-0.02	
2	RIBM		1.99	5.58		8.63	RIBM	1.58	2.04	5.51			8.63	0
	BCC	1.54	1.98	5.55	1.01901	8.59	BCC	1.58	2.03	5.54	1.02184	0.99703	8.66	0.07
	$\Delta$ RIBM – BCC		0.01	0.03		0.04	$\Delta$ RIBM – BCC	0	0.01	-0.03			-0.03	
3	RIBM		5.62	4.96		13.41	RIBM	4.36	5.56	4.82			13.22	-0.19
	BCC	4.39	5.61	4.92	1.01151	13.37	BCC	4.36	5.57	4.87	1.01918	0.99219	13.27	-0.10
	$\Delta$ RIBM – BCC		0.01	0.04		0.04	$\Delta$ RIBM – BCC	0	-0.01	-0.05			-0.05	
4	RIBM		5.84	5.24		13.99	RIBM	4.52	5.78	5.10			13.78	-0.21
	BCC	4.56	5.84	5.20	1.01234	13.95	BCC	4.51	5.77	5.17	1.02038	0.99194	13.83	-0.12
	$\Delta$ RIBM – BCC		0	0.04		0.04	$\Delta$ RIBM – BCC	0.01	0.01	-0.07			-0.05	
5	RIBM		4.94	3.93		11.46	RIBM	3.86	4.92	3.81			11.32	-0.14
	BCC	3.88	4.95	3.90	1.00831	11.44	BCC	3.85	4.91	3.87	1.01518	0.99304	11.36	-0.08
	$\Delta$ RIBM – BCC		-0.01	0.03		0.02	$\Delta$ RIBM – BCC	0.01	0.01	-0.06			-0.04	
6	RIBM		5.27	3.90		11.92	RIBM	4.11	5.23	3.80			11.78	-0.14
	BCC	4.14	5.27	3.87	1.00774	11.90	BCC	4.10	5.22	3.84	1.01508	0.99262	11.80	-0.10
	$\Delta$ RIBM – BCC		0	0.03		0.02	$\Delta$ RIBM – BCC	0.01	0.01	-0.04			-0.02	
7	RIBM		4.74	3.96		11.19	RIBM	3.72	4.74	3.90			11.15	-0.04
	BCC	3.71	4.73	3.93	1.00870	11.15	BCC	3.71	4.74	3.96	1.01552	0.99327	11.19	0.04
	$\Delta$ RIBM – BCC		0.01	0.03		0.04	$\Delta$ RIBM – BCC	0.01	0	-0.06			-0.04	
8	RIBM		4.60	3.80		10.84	RIBM	3.59	4.58	3.68			10.69	-0.15
	BCC	3.61	4.60	3.77	1.00825	10.81	BCC	3.59	4.57	3.73	1.01461	0.99348	10.73	-0.08
	$\Delta$ RIBM – BCC		0	0.03		0.03	$\Delta$ RIBM – BCC	0	0.01	-0.05			-0.04	

**Table 8** Derivation of CO<sub>2</sub> correction during operational measurement of apparent extract with a saccharometer (% m/m)

Sample number	Measurement with a saccharometer		Anton Paar analyzer
	A	B	C
1	2.75	2.60	2.56
2	3.25	3.10	3.09
3	3.10	2.95	2.95
4	3.30	3.00	3.07
5	3.15	2.90	2.86
6	3.10	2.85	2.85
7	2.76	2.56	2.62
8	2.72	2.51	2.57
average	3.02	2.81	2.82
Δ (A-B) →	0.21	A: operational measurement	
Δ (A-C) →	0.19	B: beer sample from measurement A heated to 20 °C and freed of CO <sub>2</sub> by shaking C: beer from measurement B filtered through kieselguhr on filter paper	

## 7 Acknowledgements

The author of this article would like to thank Mrs. RNDr. Jana Olšovská, Ph.D. from RIBM Prague, for providing comparative analyses of beers performed on an Anton Paar analyzer and using the distillation method.

## 8 References

- ASBC (2020). Tables Related to Determinations on Wort, Beer, and Brewing Sugars and Syrups [20.03.2020]. Available from: <https://www.asbc-net.org/Methods/Tools/Tables/Documents/TableWortBeerBrewingSugars.pdf#search=Determination%20on%20Wort%2C%20Beer%2C%20and%20Brewing%20Sugars%20and%20Syrups>
- Anton Paar (© 2007). Reference Manual, Alcolyzer Plus Beer, Beer Analyzing System, Firmware Version: V5.01
- Albl, V. (1990). Výroba piva a sladu pro učební obor biochemik-biochemička se zaměřením pro: výrobu piva a sladu (trans. Production of beer and malt for the field of study biochemist with a focus on: production of beer and malt). Institut výchovy a vzdělávání MZVŽ ČR v Praze, Prague, 363 pp. ISBN 80-7105-003-2. (Available only in Czech)
- Basařová, G., Doležalová, A., Kahler, M., Čepička, J. (1993). Pivovarsko-sladařská analytika (trans. Brewing and malting analytics). Merkanta s.r.o., Prague, 966 pp. (Available only in Czech)
- Basařová, G., Šavel, J., Basař, P., Lejsek, T. (2010). Pivovarství, Teorie a praxe výroby piva (trans. Brewing, Theory and practice of beer production). VŠCHT Praha, Prague, 863 pp. ISBN 978-80-7080-734-7. (Available only in Czech)
- Bednář, J., Štekr, K., Vančura, M. (1967). Pivovarsko-sladařské tabulky (trans. Brewing and malting tables). SNTL, Prague, 137 pp. (Available only in Czech)
- Cutaia, A.J., Reid, A., Speers, R.A. (2009). Examination of the Relationship Between Original, Real and Apparent Extracts, and Alcohol in Pilot Plant and Commercially Produced Beers. *J. Inst. Brew.* 115 (4): 318–327.
- Hardwick, W.A. (1995). Handbook of Brewing. Ed. Marcel Dekker Inc. New York, 714 s. ISBN 0-8247-8908-3.
- Hlaváček, F., Lhotský, A. (1972). Pivovarství (trans. Brewing), 2. přepracované vydání, SNTL Prague, 539 pp. (Available only in Czech)
- Lhotský, A. (1954). Technologie sladu a piva, 2. díl Výroba piva a strojní zařízení pivovarů, pomocné úkony (trans. Malt and beer technology, part 2 Beer production and brewery machinery, auxiliary operations). SNTL Prague, 423 pp. (Available only in Czech)
- Nielsen, H., Aastrup, S. (2004). Improving Tabarié's Formula. *Scandinavian Brewers' Review*, 61(4), 30–34.
- Nielsen, H. (2004). New Formulas for sharp Calculations. World Brewing Congress 2004, San Diego, California U.S.A.
- Nielsen, H., Kristiansen, A.G., Lasen, K. M., Erikstrøm, C. (2007). Balling's formula - scrutiny of a brewing dogma. *Brauwelt International*, 11, 90–93. Available from: <https://www.scribd.com/document/334496873/Balling-Formula-Scrutiny-of-a-Brewing-Dogma>
- Schmelzer, A.St., Skvrna F. (1877). Stručné pivovarnictví (trans. Brief Brewing). Redakce „Kvasu“, Prague, 223 pp. (Available only in Czech)
- Suk, J.T. (1880). Nový Poupě (trans. The New Poupě), 1. díl. Slavík & Borový, Prague, 254 pp. (Available only in Czech)
- Šavel, J. (1989). Použití výpočetní techniky v pivovarských laboratořích (trans. Use of computer technology in brewing laboratories). *Kvasny prumysl*, 35(3), 69–71. Available in Czech from: <https://www.kvasnyprumysl.cz/pdfs/kpr/1989/03/02.pdf>
- Šavel, J., Košin, P., Brož, A. (2015). Balling Alcohol Factors from the Perspective of Contemporary Brewing. *Kvasny prumysl*, 61(4), 120–128. <https://doi.org/10.18832/kp2015015>
- Šula, J. (1898). Návod ku zkoušení surovin, produktů a odpadků pivovarnických, 1. díl Rozbory chemické. Podporou „Jednoty k založení a vydržování výzkumného ústavu pro průmysl pivovarnický v Čechách“ (trans. Instructions for testing of raw materials, products and brewery waste, Part 1 Chemical analysis. With the support of the “Unity for the Establishment and Maintenance of the Research Institute of Brewing Industry in Bohemia”). Nákladem vlastním (At my own expense), Prague, 336 pp. (Available only in Czech)
- Thausing, J.E. (1882). Die Theorie und Praxis Malzberaitung und Bierfabrikation, 2. ed., J.M. Gebhardt's Verlag, Leipzig, 880 pp.